INDIANA
DEPARTMENT *of*
EDUCATION

2023 INDIANA ACADEMIC STANDARDS

MATHEMATICS

ANALYTICAL ALGEBRA II

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Indiana Academic Standards Context and Purpose

Introduction

The Indiana Academic Standards for Analytical Algebra II are the result of a process designed to identify, evaluate, synthesize, and create high-quality, rigorous learning expectations for Indiana students.

Pursuant to Indiana Code (IC) 20-31-3-1(c-d), the Indiana Department of Education (IDOE) facilitated the prioritization of the Indiana Academic Standards. All standards are required to be taught. Standards identified as essential for mastery by the end of the course are indicated with shading and an “E.” The learning outcome statement for each domain immediately precedes each set of standards.

The Indiana Academic Standards are designed to ensure that all Indiana students, upon graduation, are prepared with essential knowledge and skills needed to access employment, enrollment, or enlistment leading to service.

What are the Indiana Academic Standards and how should they be used?

The Indiana Academic Standards are designed to help educators, parents, students, and community members understand the necessary content for each course, and within each content area domain, to access employment, enrollment, or enlistment leading to service. These standards should form the basis for strong core instruction for all students at each grade level and content area. The standards identify the minimum academic content or skills that Indiana students need in order to be prepared for success after graduation, but they are not an exhaustive list.

While the Indiana Academic Standards establish key expectations for knowledge and skills and should be used as the basis for curriculum, the standards by themselves do not constitute a curriculum. It is the responsibility of the local school corporation to select and formally adopt curricular tools, including textbooks and any other supplementary materials, that align with Indiana Academic Standards. Additionally, corporation and school leaders should consider the appropriate instructional sequence of the standards as well as the length of time needed to teach each standard. Every standard has a unique place in the continuum of learning, but each standard will not require the same amount of time and attention. A deep understanding of the vertical articulation of the standards will enable educators to make the best instructional decisions. These standards must also be complemented by robust, evidence-based instructional practices to support overall student development. By utilizing strategic and intentional instructional practices, other areas such as STEM and employability skills can be integrated with the content standards.

Content-Specific Considerations

The Indiana Academic Standards for Analytical Algebra II consist of six domains: Arithmetic and Structure of Expressions, Equations, and Functions; Function Families; Modeling with Functions and Data; Modeling with Advanced Algebra; Modeling with Data and Statistics; and Modeling with Quantities. The skills listed in each domain indicate what students should know and be able to do by the end of the course. The Process Standards demonstrate the ways in which students should develop conceptual understanding of mathematical content, and the ways in which students should synthesize and apply mathematical skills.

Acknowledgments

The Indiana Department of Education appreciates the time, dedication, and expertise offered by Indiana's K-12 educators, higher education professors, representatives from business and industry, families, and other stakeholders who contributed to the development of the Indiana Academic Standards. We wish to specially acknowledge the committee members, as well as participants in the public comment period, who dedicated many hours to the review and evaluation of these standards designed to prepare Indiana students for success after graduation.

Mathematics Process Standards

PS.1: Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway, rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” and "Is my answer reasonable?" They understand the approaches of others to solving complex problems and identify correspondences between different approaches. Mathematically proficient students understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
PS.2: Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
PS.3: Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They analyze situations by breaking them into cases and recognize and use counterexamples. They organize their mathematical thinking, justify their conclusions and communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. They justify whether a given statement is always true, sometimes, or never. Mathematically proficient students participate and collaborate in a mathematics community. They listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
PS.4: Model with mathematics.
Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace using a variety of appropriate strategies. They create and use a variety of representations to solve problems and to organize and communicate mathematical ideas. Mathematically proficient students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

PS.5: Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Mathematically proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. Mathematically proficient students identify relevant external mathematical resources, such as digital content, and use them to pose or solve problems. They use technological tools to explore and deepen their understanding of concepts and to support the development of learning mathematics. They use technology to contribute to concept development, simulation, representation, reasoning, communication, and problem solving.
PS.6: Attend to precision.
Mathematically proficient students communicate precisely to others. They use clear definitions, including correct mathematical language, in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They express solutions clearly and logically by using the appropriate mathematical terms and notation. They specify units of measure and label axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and check the validity of their results in the context of the problem. They express numerical answers with a degree of precision appropriate for the problem context.
PS.7: Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. They step back for an overview and shift perspective. They recognize and use properties of operations and equality. They organize and classify geometric shapes based on their attributes. They see expressions, equations, and geometric figures as single objects or as being composed of several objects.
PS.8: Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated and look for general methods and shortcuts. They notice regularity in mathematical problems and their work to create a rule or formula. Mathematically proficient students maintain oversight of the process, while attending to the details as they solve a problem. They continually evaluate the reasonableness of their intermediate results.

Analytical Algebra II

Standards identified as essential for mastery by the end of the course are indicated with gray shading and an "E." The learning outcome statement for each domain immediately precedes each set of standards.

Arithmetic and Structure of Expressions, Equations, and Functions	
Learning Outcome: Students simplify, manipulate, and solve nonlinear expressions, equations, and functions in a variety of forms.	
AAII.ASE.1	Explain how extending the properties of integer exponents to rational numbers allows for a notation for radicals in terms of rational exponents (e.g., $5^{1/3}$) and explain how this is defined.
AAII.ASE.2	Rewrite algebraic rational expressions in equivalent forms (e.g., using properties of exponents and factoring techniques) and describe how rewriting those expressions reveals mathematical structure. Add, subtract, multiply, and divide algebraic rational expressions.
AAII.ASE.3	Solve systems of equations consisting of linear and nonlinear equations or functions in two variables algebraically and graphically.
Function Families	
Learning Outcome: Students represent nonlinear functions in a variety of forms, recognizing and applying key features based on the type of function.	
AAII.FF.1	Using technology, identify, create, and connect algebraic and graphical representations of each of the function families listed. Model real-world situations with each function family: a. Quadratic b. Polynomial c. Square root d. Rational e. Exponential f. Piecewise-defined and absolute value functions (E)
AAII.FF.2	Graph each of the families of function with and without technology. Identify and describe key features, such as intercepts, domain and range, asymptotes, symmetry, and end behavior. Create inverse functions algebraically and/or graphically based on a given function.
AAII.FF.3	Use graphical and algebraic structures and techniques to transform functions into equivalent forms to expose different information and identify key features. Connect the meaning of the key features to contextual situations.
AAII.FF.4	Solve real-world problems with each function family, including situations in the context of science and economic phenomena. (E)

Modeling with Functions and Data	
Learning Outcome: Students represent real-world situations with linear and nonlinear functions, and use these equations to solve problems.	
AAII.MFD.1	Define functions and their inverses and illustrate examples algebraically and graphically. Identify real-world situations that can be modeled using functions. (E)
AAII.MFD.2	Represent real-world problems that can be modeled by linear, quadratic, exponential, and rational functions using tables, graphs, and equations. Use technology to represent the functional relationships and translate and interpret different forms (e.g., vertex form of a quadratic, intercepts, end behavior) with respect to the context. (E)
AAII.MFD.3	Use technology to find a linear, quadratic, or exponential function that models a relationship for a bivariate data set to make predictions; interpret the correlation coefficient for linear models. Compare and evaluate model fit using different function families. (E)
AAII.MFD.4	Explore the effects of function transformations using graphing technology. Explain the effects of transformations of functions such as $f(x) + k$, $kf(x)$, $f(kx)$, or $f(x + k)$ for different functions and values of k.
Modeling with Advanced Algebra	
Learning Outcome: Students use advanced algebra concepts to model real-world function situations and use specific algebraic techniques to reveal and make use of structure with families of functions.	
AAII.MAA.1	Use algebraic and graphical strategies to make use of structure with quadratic, polynomial, and rational functions to solve real-world problems, including but not limited to: a. Completing the square to rewrite contextual quadratic functions in vertex form and interpret the outcome; b. Determining the number of solutions to a function using graphical and algebraic forms (including the discriminant and complex numbers as appropriate); c. Factoring, chunking, and rewriting functions using properties of exponents; and d. Identifying and explaining extraneous roots.
AAII.MAA.2	Represent and solve real-world systems of linear equations and inequalities in two or three variables algebraically and using technology. Interpret the solution, and determine whether it is reasonable.
AAII.MAA.3	Model real-world phenomena using linear programming and matrices.
Modeling with Data and Statistics	
Learning Outcome: Students use statistics and probability techniques to collect and interpret complex data that can be modeled using functions.	
AAII.MDS.1	Distinguish between random and non-random sampling methods, identify possible sources of bias in sampling, describe how such bias can be controlled and reduced, evaluate the characteristics of a good survey and well-designed experiment, design simple experiments or investigations to collect data to answer questions of interest, and make inferences from sample results. (E)
AAII.MDS.2	Using the results of a simulation, decide if a specified model is consistent with the results. Construct a theoretical model, and apply the law of large numbers to show the relationship between the two models.

AAII.MDS.3	Use data science techniques such as predictive modeling, linear algebra, and conditional probability to analyze data sets and make and evaluate claims.
Modeling with Quantities	
Learning Outcome: Students use combinatorics to quantify and model real-world situations.	
AAII.MQ.1	Using technology, model real-world probability situations using permutations, combinations, and the Fundamental Counting Principle. (E)